

How Does Money Grow Over Time?

Lesson Summary

How Does Money Grow Over Time? will teach students about types of interest and various savings venues.

Lesson Objectives

- Define compound interest and explain the effect of compounding interest on a daily, monthly, quarterly, or annual basis.
- Investigate various investment and saving opportunities.
- Define and demonstrate comprehension of the following terms: saving, investing, Rule of 72, compound interest, and diversification.

NCTM Standards

1A - Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

1B - Understand meanings of operations and how they relate to one another.

1C - Compute fluently and make reasonable estimates.

2A - Understand patterns, relations, and functions.

5A - Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

5B - Select and use appropriate statistical methods to analyze data.

5C - Develop and evaluate inferences and predictions that are based on data.

5D - Understand and apply basic concepts of probability.

6B - Solve problems that arise in mathematics and in other contexts.

6D - Monitor and reflect on the process of mathematical problem solving.

8A - Organize and consolidate mathematical thinking through communication.

8B - Communicate mathematical thinking coherently and clearly to peers, teachers, and others.

9C - Recognize and apply mathematics in contexts outside of mathematics.

Mathematical Strands

	Thinking Algebraically	Students will use the formula $FV = P(1+r)^t$ to calculate the value of future investments.	
	Interpreting Statistics	Students will calculate the percentage returns from investments in the stock market and then compare those rates of return to the interest rates money could have been earning in a bank. Students will calculate the rate of return of the Dow Jones Industrial Average.	
	Communicating Quantitative Information	Students will practice their skills with compound interest, calculate investment growth, and graph exponential growth.	
	Tackling Complex Problems	Students will calculate compound interest over a long period of time to witness the dramatic growth produced by compounded interest. You may choose to tell them the formula: Future Value = Principal · (1+interest rate) ^{time} or $FV=P(1+r)^t$. This is also an opportunity to teach students to use formulas in a spreadsheet application (such as Microsoft Excel or Lotus 1-2-3).	

THINKING ALGEBRAICALLY

Calculating the Value of Future Investments

Use the formula presented to find the value of the missing information.

$FV = P(1+r)^t$, where FV is the future value of an investment

P is the principal or initial investment

r is the interest rate, expressed as a decimal

t is the time, expressed in years.

	Principal	Interest Rate	Years	Future Value
1.	\$3,000	8%	10	
2.	\$1,500	10%	5	
3.	\$7,000,000	2%	2	
4.	\$20	3%	60	
5.	\$804	1%	15	
6.	\$382	11%	20	
7.	\$4,560	4%	7	
8.	\$0.01	5%	500	
9.	\$30,000,000	5%	2	
10.		6%	10	\$24,000
11.		4%	100	\$5,000,000
12.		3%	2	\$34,290



INTERPRETING STATISTICS

Below is a table listing yearly close data of the Dow Jones Industrial Average from 1997 to 2006.

(Dow Jones Indexes. January 2007 <http://www.djindexes.com/mdsidx/downloads/xlspages/DJIA_Hist_Perf.xls>)

Calculate the rate of return for each one-year period. Use the following formula:

$$\text{Rate of return} = \frac{(\text{year's close}) - (\text{prior year's close})}{\text{prior year's close}}$$

Trade	Year's Close	Rate of Return
December 1997	7,908.25	-----
December 1998	9,181.43	16.10%
December 1999	11,497.12	25.22%
December 2000	10,787.99	-6.17%
December 2001	10,021.57	-7.10%
December 2002	8,341.63	-16.76%
December 2003	10,453.92	25.32%
December 2004	10,783.01	3.15%
December 2005	10,717.50	-0.61%
December 2006	12,463.15	16.29%

1. For which year was the rate of return from the Dow Jones the greatest?
2. For which year was the rate of return from Dow Jones the smallest?
3. Some of the rates of return from the Dow Jones Industrial Average are small and some are large. What would your advice be to someone who saw these rates of return and decided to invest *all* of their savings in stocks? Is this the best investment idea? Why or why not?

This is a table of the federal interest rate for the same years.

Board of Governors of the Federal Reserve System. January 2007 <www.federalreserve.gov>

Year	Interest Rate
1997	8.44%
1998	8.35%
1999	8.00%
2000	9.23%
2001	6.91%
2002	4.67%
2003	4.12%
2004	4.34%
2005	6.19%
2006	7.96%

Based on the DJIA table and the interest rate table to the left, in which years would it have been better to invest some money in the stock market rather than all the money in the bank? Why?



COMMUNICATING QUANTITATIVE INFORMATION

Investment and Compounded Interest

Becca and David are trying to put money aside for their children's education. Their first child will be attending college in 10 years and they have about \$11,500 saved so far. They plan on saving another \$2,000 a year to put into this account each January.

Assume that Becca and David's investment account has an annually compounded interest rate of 6%. They want to know how much money they will have when their first child starts college.

1. Complete the table below by calculating the total value of their investment account for years 6-10.

Year	Value of Investment	Value after Additional Deposit	Interest	New Amount of Investment
	\$11,500	\$11,500	\$690.00	\$12,190.00
1	\$ 12,190.00	\$14,190.00	\$851.40	\$15,041.40
2	\$ 15,041.40	\$17,041.40	\$1,022.48	\$18,063.88
3	\$18,063.88	\$20,063.88	\$ 1,203.83	\$21,267.71
4	\$21,267.71	\$23,267.71	\$1,396.06	\$24,663.77
5	\$24,663.77	\$26,663.77	\$1,599.83	\$28,263.60

2. You are their financial planner and you will be meeting with the couple to show them how this college fund will grow. For your meeting you will need to give a brief talk (2-4 minutes). Prepare a graph or table to explain to Becca and David their investment's projected growth.



TACKLING COMPLEX PROBLEMS

The Importance of Time in Investing

If Peter invests \$100 at the beginning of each year, starting at age 15 and ending at age 65, in a simple savings account that has an annually compounded interest rate of 4%, how much money will be in the account when he is 65?

A table showing the value of Peter's investment has been started for you below.

Year	New Value of Investment	Value of Investment After Deposit	Interest Earned
15	\$0.00	\$100.00	\$4.00
16	\$104.00	\$204.00	\$8.16
17	\$212.16	\$312.16	\$12.49
18	\$324.65	\$424.65	\$16.99
19	\$441.63	\$541.63	\$21.67
...			
52	\$8,497.03	\$8,597.03	\$343.88
53	\$8,940.91	\$9,040.91	\$361.64
54	\$9,402.55	\$9,502.55	\$380.10
55	\$9,882.65	\$9,982.65	\$399.31
56	\$10,381.96	\$10,481.96	\$419.28
57	\$10,901.24	\$11,001.24	\$440.05
58			
59			
60			
61			
62			
63			
64			
65			

1. Explain how the *Value of the Investment After Deposit* was calculated in year 2.
2. Explain how the *Interest Earned* was calculated in year 2.
3. Explain how the *New Value of Investment* was calculated in Year 3.
4. Using your knowledge of the table, complete the last eight rows.
5. If Rachel, Peter's friend, decides at age 30 that she would like to start a savings account that would have as much money in it as Peter's does when they are both 65, how much money would she need to invest all at once at age 30?

Use the formula: $I = P(1+r)^t$, where I is the value of the initial investment (P), after being invested for t years at an rate of return of r .

